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EXPERIMENTAL AUCTION MARKETS AND THE WALRASIAN HYPOTHESIS[1]

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This study reports on a block of experimental market sessions designed primarily to provide (1) the severest test yet attempted of the equilibrating forces operating in competitive auction markets and (2) a more rigorously controlled test of the Walrasian hypothesis.[2] Some data are also supplied which show the effect of cash payoffs on the equilibrating behavior of such markets; in particular, the effect of full cash payoffs to all successful trading subjects as against payoffs to a subset of such subjects chosen at random.

EXPERIMENTAL DESIGN AND SUBJECTS

The supply and demand conditions underlying the experimental design in this study were intentionally unconventional. In each experimental session, each of eleven subject buyers could purchase at most one unit of the fictitious commodity per trading period at a price not to exceed the limit price \$4.20. Therefore, the demand per unit of time, or trading period, was perfectly elastic at \$4.20 up to the maximum demand quantity of eleven units. In each session each subject seller could sell at most one unit of the commodity at any price not below the given minimum reservation price \$3.10. There were thirteen such sellers in two experimental sessions, sixteen in two additional experimental sessions, and nineteen in the final two markets. Therefore, the supply per trading period was perfectly elastic up to the maximum supply quantities of thirteen, sixteen, and nineteen units, respectively, in the three experimental treatments.

Each session was begun with a general statement that the group was being asked to participate in a decision-making experiment; that they would not be subjected to any unpleasant stimuli or experiences; and, furthermore, that they would have an opportunity to earn real money during their participation. Copies of instructions were passed out and read aloud to the entire group.[3] The payoff formula for each buyer in each trading period was \$0.05 for making a contract plus the difference between his limit buy price and his contract price. Each seller received \$0.05 for making a contract plus the difference between his contract price and his limit sell price.

Each subject trader had initial information only on his own limit price.[4] The additional information provided in the course of the market sessions consisted of the ordered public bids and offers announced by the individual traders. Since the public acceptance of a bid or offer constituted a contract, each trader knew which bids and offers were

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[2] An earlier paper (V. L. Smith, "An Experimental Study of Competitive Market Behavior," *Journal of Political Economy*, LXX [April, 1962], 126–34) seemed to provide sufficient evidence to warrant the tentative conclusion that a linear version of the well-known Walrasian hypthesis of competitive market-adjustment behavior was inferior to a linear test alternative called the "excess-rent hypothesis."

[3] The printed instructions given to each subject were reproduced in the Appendix to V. L. Smith, "Effect of Market Organization on Competitive Equilibrium," *Quarterly Journal of Economics*, LXXXVII (May, 1964), paragraphs 1–4, 5SB, 6SB, 199–201.

[4] The same subjects held the same limit price cards in all trading periods, and this fact was evident to all the subjects. They did not know that all buyers had the same limit prices, and all sellers had the same limit prices. They were not told the number of buyers or the number of sellers, but they soon became aware, in successive trading periods, that there was excess supply at the end of each trading period.

accepted and which were not. Under these information conditions it is difficult to imagine a test of the equilibrium tendencies in auction markets that would be more severe than the design described above. In equilibrium, with these supply and demand conditions, the entire rent in this market, (\$4.20 - \$3.10)11 = \$12.10 per trading period, must be allocated to buyers! In full competitive equilibrium, each seller would receive only his "normal profit" commission of \$0.05 per transaction, while buyers receive \$1.15 per transaction. Under such cash-payoff conditions one would expect sellers to be very resistant to contract prices being forced down to the \$3.10 equilbrium. Thus, suppose each subject is assumed to have a utility function for additional income which is concave from below, and that an individual's bargaining resistance is proportional to marginal utility. Then the nearer is price to the \$3.10 equilibrium, the greater is seller resistance to a further reduction in price and the weaker is buyer resistance to an increase in price. The question is whether the competition created by excess sellers will produce equilibrium even under this condition of exaggerated imbalance in the rental rewards to bargaining.

The 162 subjects participating in these experiments were Sophomore and Junior students enrolled in three sections each of two undergraduate courses in economics. One course was introductory economics, the other introductory economic theory. Two replications were run under each of the three values, 2, 5, and 8 for the "treatment" variable, e = excess supply. Table 1 illustrates the over-all experimental design and indicates the combination of experimental conditions, course, and number of subjects associated with each session. No subject participated in more than one of the sessions. The sessions were run separately in each of two series separated by several months. Subjects were given no advance warning that an experiment was going to be performed in their class, and the experimental sessions discussed in this paper were intermingled with sessions for entirely different experiments. This procedure was used to minimize information transfer between subject groups.[5]

TABLE 1

NUMBER OF SUBJECTS AND EXPERIMENTAL CONDITION FOR EACH SESSION

Experimental Session No.	Condition			
	$e=2$	$e=5$	$e=8$	Total Subjects
Course 1	1 ($N=24$)	3 ($N=27$)	5 ($N=30$)	81
Course 2	2 ($N=24$)	4 ($N=27$)	6 ($N=30$)	81
Total subjects	48	54	60	162

EXPERIMENTAL DATA

Figures 1*A*–1*F*, corresponding to experimental sessions 1–6, provide complete series of contract prices in the order in which they were executed in the four trading periods of each session (six trading periods of session 1). Cash payoffs for the six sessions totaled \$342 of which \$281.70 represented the earnings of buyers, the remainder being earned by sellers. Motivation was excellent.

In spite of the extreme asymmetry in buyer and seller rent, it is seen that contract prices show a strong tendency to converge to the theoretical competitive equilibrium. It is also clear from these charts that the tendency to equilibrium is an increasing function of e (a precise measure of this tendency is discussed later in Table 2). In sessions 1 and 2 (e = 2), only six contracts were at equilibrium in trading periods 1–4;

[5] See Smith, "Effect of Market Organization on Competitive Equilibrium," *op. cit.*, pp. 184–86.

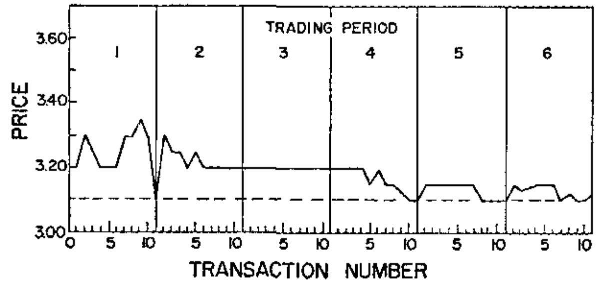

Fig. 1*A*.—Experimental session 1, $e = 2$

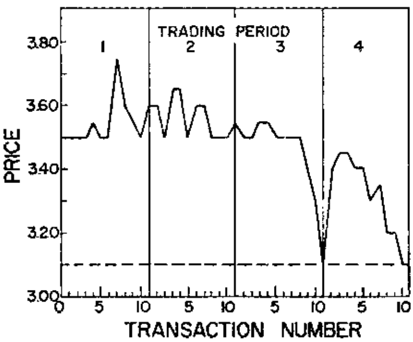

Fig. 1*B*.—Experimental session 2, $e = 2$

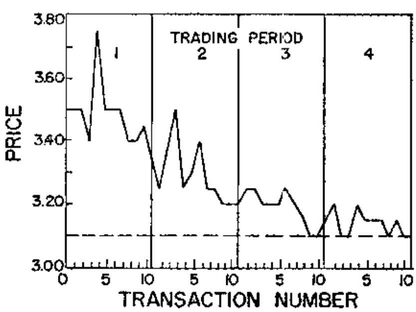

Fig. 1*C*.—Experimental session 3, $e = 5$

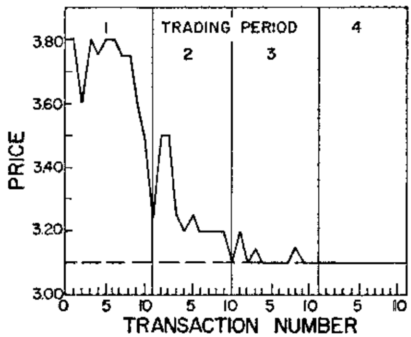

Fig. 1*D*.—Experimental session 4, $e = 5$

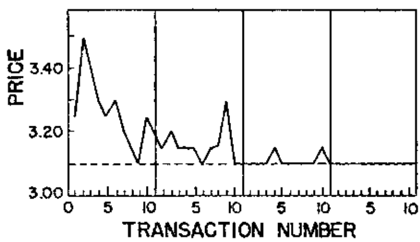

Fig. 1*E*.—Experimental session 5, $e = 8$

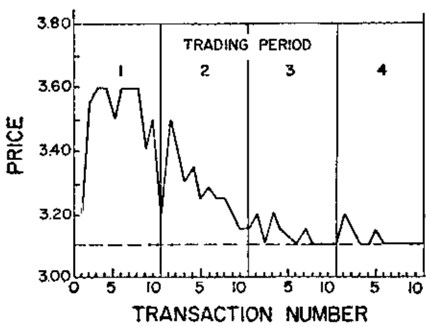

Fig. 1*F*.—Experimental session 6, $e = 8$

in sessions 3 and 4 ($e = 5$), twenty-seven contracts were at equilibrium; in sessions 5 and 6 ($e = 8$) there were thirty-seven such contracts. Two additional trading periods were run in experiment 1 to see if the added trading experience would produce equilibrium under the weak equilibrating condition, $e = 2$. From Figure 1*A* the convergence tendencies continue in evidence, though weakly, in trading periods 5 and 6.

Session 2 shows the degree to which it may be possible for sellers to maintain prices above equilibrium for comparatively long series of transactions when there are only two excess sellers in the market. In this session a form of temporary tacit co-operation among sellers produced a minimum price line of \$3.50 until transaction 9 in period 3. This co-operative set was then broken by the seller who sold at \$3.75 (yielding a cash payoff of \$0.70) in period 1 and then failed to make a contract in period 2. Rather than fail again in period 3, he sold at \$3.40. This, of course, alerted all buyers to the possibility of making contracts below \$3.50, with the result that all remaining contracts in periods 3 and 4 were at prices below \$3.50. In the absence of formal collusion and side payments, which were prohibited in these sessions, such tacit co-operation is extremely difficult to maintain. One "nervous" seller may be sufficient to break the co-operative set, and the probability of having at least one increases with e. Thus, with $e = 5$ and $e = 8$ in sessions 3-6, we see no such extensive "price lines" being established above \$3.10.

From Figures 1*A* and 1*B* one might be tempted to conjecture that the initial contract price was highly significant in determining the course of the market. The initial contract in experiment 1 was at \$3.20, and thereafter most contract prices were not far from this level. Experiment 2 began at \$3.50 and did not fall below this level for some time. But experiment 6, as it happens, also began at \$3.20 and rose much above this level for several transactions in spite of the large excess supply of eight units. Similarly, experiment 3 began at \$3.50 but prices behaved differently than in experiment 2. First-period contracts tend to be very erratic and sensitive to subtle differences in the dynamics of different subject groups. The main trends that can be related to more traditional economic variables emerge more clearly after the first-period learning experience is completed.

TESTS OF THE WALRASIAN HYPOTHESIS

The major analytical purpose of this paper is to test the Walrasian hypothesis (WH) against a test alternative, the excess rent hypothesis (ERH). As we use them here, WH refers to the hypothesis that price tends to fall (rise) at a rate which is proportional to the excess supply (demand) at any given price and ERH refers to the hypothesis that price tends to fall (rise) at a rate proportional to excess economic rent at any given price, where excess rent is measured by the area between the supply and demand curves from the price in question down (or up) to the equilibrium price. In the present design, at the price p_t and an excess supply e, excess rent is $e\,(p_t - 310)$. The significance of this design is that e becomes a design constant under experimental control at all feasible contract prices. Furthermore, e is independent of which particular buyers and sellers are paired in each contract.

We should note that no a priori commitment to ERH is intended. It is WH which has a long, and by frequency of reference, perhaps a distinguished history. You cannot test any hypothesis except by reference to a competing test alternative, and ERH represents such a plausible alternative. In this experimental design, ERH turns out to have a distinct intuitive appeal. To see this, imagine price being temporarily "established" at p_t. If this price were to persist in further contracts, any seller failing to make a contract at p_t stands to forego a profit (rent) equal to $p_t - 310$ cents. If $e = 2$, so that we must have two sellers failing to make contracts, the total potential loss at p_t is $2(p_t - 310)$. Under ERH the assumption is that price-cutting occurs at a rate proportional to this potential monetary loss and is influenced both by the number of excess sellers *and* their individual potential losses. Thus, under ERH, the rate of price-cutting diminishes as p_t falls and the potential loss decreases. In other words, if you have a lot to lose by failing to make a contract, you are quick to undercut your competitors in order to increase your chances of making a sale; if you have little to lose by failing to make a contract, you are slower to undercut your competitiors. Under WH, price-cutting is independent of such potential trading losses, depending only on the constant excess supply on the market.

Mathematically, WH implies an adjustment equation of the following form:

$$\Delta p_{t+1} = p_{t+1} - p_t = \begin{cases} \beta e + u_{t+1}, \text{ if } p_t > p^0 - \beta e \\ u_{t+1}, \text{ if } p^0 \leq p_t \leq p^0 - \beta e, \beta < 0, \end{cases} \quad (1)$$

while ERH implies

$$\Delta p_{t+1} = p_{t+1} - p_t = ae(p_t - p^0) + v_{t+1}, \quad a < 0, \quad (2)$$

where $p^0 = 310$ is the theoretical equilibrium in cents, β is the adjustment speed coefficient under WH, a is the adjustment speed coefficient under ERH, and p_t is the contract price on the tth transaction. With this experimental design it is evident that the two hypotheses have quite distinct empirical implications. Phase lines for each of the two hypotheses are shown in Figure 2 for two levels of the control variable, e_1 and e_2, where $e_1 < e_2$. Under WH the phase lines all have a slope of unity, and increases in e simply shift these lines parallel to the right. Under ERH, the phase line has a slope less than unity and this slope is a decreasing function of e as shown. It is seen that our peculiar experimental supply and demand design provides a relatively crucial test of WH as opposed to ERH.

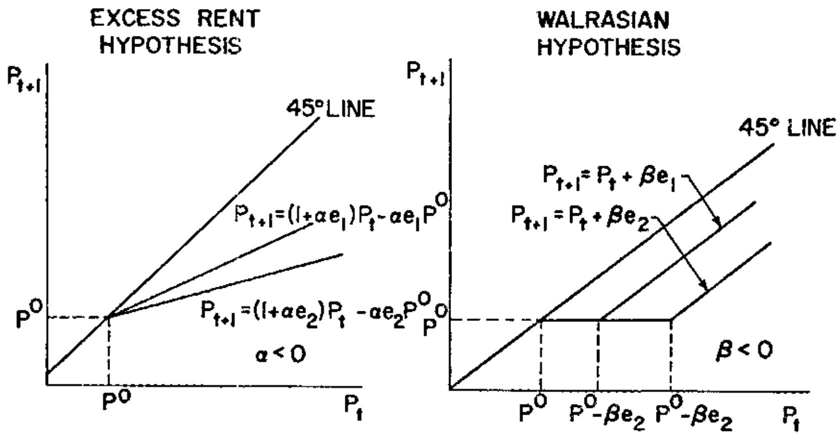

FIG. 2

Using this analysis and the data from the six experiments reported in the last section the test of WH against ERH is based upon the stochastic process defined by

$$\Delta p_{t+1} = p_{t+1} - p_t = a_{01} + a_1(p_t - p^0)e + \beta_1 e + \epsilon_{1,\,t+1}, \quad (3)$$

which is a general linear hypothesis containing both WH and ERH as special polar cases.

Table 2 shows the results of least-squares regression estimates of the coefficients of equations (1)–(3). Because of the "kink" in the phase line at $p_t = p^0 - \beta e$ (see Fig. 2) implied by WH, the regressions were performed using all the observations from the six experiments and again with the observations $0 \leq \hat{p}_t < 0.05$ omitted. The second regression reduces bias in favor of ERH by eliminating observations in the flat region of the kink. Under classical significance tests we see in either case, from the standard errors of $\hat{\alpha}_1$ and $\hat{\beta}_1$, that $\hat{\alpha}_1$ is quite significantly different from zero, whereas $\hat{\beta}_1$ is not.[6]

EFFECT OF CASH REWARDS ON CONVERGENCE

Before conducting the six experiments discussed above, two pilot experimental sessions were run. In the pilot sessions instead of paying every buyer (seller) who made a contract the difference between his contract price and his limit price plus \$0.05, this amount was paid to four subjects selected at random at the end of each trading period. This reinforcement formula was made known to the subjects at the beginning of each of the pilot sessions. Expected rewards in these sessions would, of course, be much lower than with full cash payoffs to all trading subjects. The objective was to provide a low-cost means of testing the mechanics of the experimental technique prior to performing the six analysis sessions and to provide two control sessions with weak payoffs to determine the effect of reinforcement condition on convergence.

Table 3 provides least-squares estimates of the parameters of equation (4):

$$\pi_{t+1} = \alpha_0 + \beta_0 \pi_t + \epsilon_{t+1}, \qquad (4)$$

where $\pi_t = p_t - p^0$. If we define

$$\lim_{t \to \infty} E(\pi_t) = E(\pi_\infty),$$

as the expected deviation in experimental equilibrium price from the theoretical equilibrium, then it is readily shown that[7] $E(\pi_\infty)$

TABLE 2

REGRESSION ESTIMATES: WH VERSUS ERH

Observation Subsets	$p_{t+1} - p_t = \alpha_{01} + \alpha_1(p_t - p^0)e + \beta_1 e + \epsilon_{1,t+1}$*			$p_{t+1} - p_t = \alpha_{02} + \alpha_2(p_t - p^0)e + \epsilon_{2,t+1}$		$p_{t+1} - p_t = \alpha_{03} + \beta_3 e + \epsilon_{3,t+1}$	
	$\hat{\alpha}_{01}$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\alpha}_{02}$	$\hat{\alpha}_2$	$\hat{\alpha}_{03}$	$\hat{\beta}_3$
With all observations. ($N = 259$)	−0.6134 (1.108)	−0.0226 (0.0051)	0.2198 (0.1952)	0.3419 (0.597)	−0.0213 (0.0050)	−1.3317 (0.507)	0.0258 (0.1972)
With $0 \leq \hat{p}_t < 0.05$ omitted.... ($N = 189$)	−0.7216 (1.618)	−0.0204 (0.0073)	0.1595 (0.3008)	−0.2191 (0.873)	−0.0185 (0.0064)	−0.9868 (1.328)	−0.2469 (0.2679)

* Standard errors are shown in parentheses.

[6] Bayesian Report: If we assume a uniform joint prior distribution of α_1, β_1, μ, and $\log \sigma^2$, where $\mu = \alpha_{01} + \alpha_1(p_t - p^0)e + \beta_1 e$, in equation (3), then, under the assumptions of normal regression theory, the joint posterior distribution of α_1 and β_1 is bivariate normal, conditional on σ^2 with parameters

$$\hat{\alpha}_1 = -0.0204\,, \qquad \hat{\beta}_1 = 0.1595\,,$$

$$\sigma(\hat{\alpha}_1) = 0.073\,, \qquad \sigma(\hat{\beta}_1) = 0.3008\,,$$

$$\sigma = 0.2109\,,$$

$$\text{for the regression with } N = 189\,.$$

From these parameters and tables of ordinates and cumulative probabilities for the normal distribution we compute, using Bayes's theorem, the following posterior experimental odds favoring ERH as against WH:

$$\frac{P(\text{ERH})}{P(\text{WH})} = \frac{P\left(\frac{2}{e} < \alpha_1 < 0,\, \beta_1 = 0\right)}{P(\beta_1 < 0,\, \alpha_1 = 0)}$$

$$= \frac{P\left(-\frac{2}{e} < \alpha_1 < 0 \,\middle|\, \beta_1 = 0\right) P(\beta_1 = 0)}{P(\beta_1 < 0 \mid \alpha_1 = 0) P(\alpha_1 = 0)} > 300\,.$$

The odds favoring ERH are over 300 to 1.

[7] Cf. "Effect of Market Organization," *op. cit.*, p. 194.

$= \alpha_0/(1 - \beta_0)$. Estimates of $E(\pi_\infty)$ are also contained in Table 3. It is seen that under the full payoff condition the experimental market equilibrium is only 4.5 cents below the theoretical, for $e = 2$, and 4.3 cents above it for $e = 5$, as compared with a discrepancy of 26.4 and 13.8, respectively, under weak payoffs. A t-test on the $\hat{\alpha}_0$ for weak payoffs shows $\hat{\alpha}_0$ to be significantly above zero for $e = 2$, $t(2) = 1.95$, but not for $e = 5$, $t(5) = 0.99$. With full payoffs under neither condition of excess supply is $\hat{\alpha}_0$ significantly different from zero, $t(2) = -0.087$, $t(5) = 0.40$. An F-test comparison of the estimates $\hat{\sigma}^2$ under weak and full cash payoffs shows the differences to be highly significant ($\alpha < 0.005$) for $e = 2$, but insignificant for $e = 5$. We conclude that there exist some conditions under which experimental results are likely to be biased to an important degree by the substitution of random for full cash payoffs. Consequently, the use of random payoffs cannot generally be defended as a compromise between no payoffs and full cash payoffs.

TABLE 3

COMPARISON OF FULL VERSUS WEAK (RANDOM) PAYOFFS

Excess Supply, E	Experiment	$\hat{\alpha}_0$*	$\hat{\beta}_0$	$\hat{\sigma}^2$	No. Observations	$E(\pi_\infty)$
2	1, 2; full payoff	−0.188 (2.435)	0.9584 (0.0367)	6.207	100	−4.52
	A; weak payoff	5.753 (2.960)	0.7820 (0.0939)	12.885	58	26.4
5	3, 4; full payoff	0.528 (1.329)	0.8769 (0.0461)	8.801	79	4.29
	B; weak payoff	2.079 (2.110)	0.8491 (0.0676)	8.455	50	13.8

* Standard errors are shown in parentheses.

SUMMARY

The results of our six experimental sessions tend to support the view that the auction-market mechanism produces strong competitive equilibrating tendencies, even under conditions of extreme imbalance in the rental rewards to buyers as opposed to sellers. These tendencies are weakest when excess supply is small, strongest when excess supply is large. This conclusion, and the results on which it is based, assume the information conditions under which our experimental markets were operated and should not be assumed without further inquiry to hold under different information conditions.[8]

A test of WH as against ERH yields strong support for the latter. The credibility of this conclusion is strengthened by the fact that the experimental design was determined by the objective of providing good discrimination between the competing hypotheses.

The experimental sessions under full cash payoffs to all subjects were compared with two pilot sessions under full cash payoffs to only a subset of subjects chosen at random. The results show enough difference in market behavior to suggest that one should not arbitrarily substitute random payoff rewards for full payoff rewards, on the assumption that the results will not be significantly altered.

[8] See L. Fouraker and S. Siegel, *Bargaining Behavior* (New York: McGraw-Hill Book Co., 1963), pp. 142–51, 184–93, for a discussion of the effect of amount of information on oligopoly bargaining behavior.